

nag_fft_complex (c06ecc)

1. Purpose

nag_fft_complex (c06ecc) calculates the discrete Fourier transform of a sequence of n complex data values.

2. Specification

```
#include <nag.h>
#include <nagc06.h>
```

```
void nag_fft_complex(Integer n, double x[], double y[], NagError *fail)
```

3. Description

Given a sequence of n complex data values z_j , for $j = 0, 1, \dots, n-1$, this function calculates their discrete Fourier transform defined by

$$\hat{z}_k = a_k + ib_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} z_j \exp\left(-i \frac{2\pi jk}{n}\right), \quad \text{for } k = 0, 1, \dots, n-1.$$

(Note the scale factor of $1/\sqrt{n}$ in this definition.)

To compute the inverse discrete Fourier transform defined by

$$\hat{w}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} z_j \exp\left(+i \frac{2\pi jk}{n}\right), \quad \text{for } k = 0, 1, \dots, n-1,$$

this function should be preceded and followed by calls of **nag_conjugate_complex (c06gcc)** to form the complex conjugates of the z_j and the $\hat{z}_k$.

The function uses the Fast Fourier Transform algorithm (Brigham 1974). There are some restrictions on the value of n (see Section 4).

4. Parameters

n

Input: the number of data values, n .

Constraint: $n > 1$. The largest prime factor of **n** must not exceed 19, and the total number of prime factors of **n**, counting repetitions, must not exceed 20.

x[n]

Input: **x[j]** must contain x_j , the real part of z_j , for $j = 0, 1, \dots, n-1$.

Output: the real parts a_k of the components of the discrete Fourier transform. a_k is contained in **x[k]**, for $k = 0, 1, \dots, n-1$.

y[n]

Input: **y[j]** must contain y_j , the imaginary part of z_j , for $j = 0, 1, \dots, n-1$.

Output: the imaginary parts b_k of the components of the discrete Fourier transform. b_k is contained in **y[k]**, for $k = 0, 1, \dots, n-1$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_C06_FACTOR_GT

At least one of the prime factors of **n** is greater than 19.

NE_C06_TOO_MANY_FACTORS

n has more than 20 prime factors.

NE_INT_ARG_LE

On entry, **n** must not be less than or equal to 1: **n** = $\langle \text{value} \rangle$.

6. Further Comments

The time taken by the function is approximately proportional to $n \log n$, but also depends on the factorization of n . The function is somewhat faster than average if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

On the other hand, the function is particularly slow if n has several unpaired prime factors, i.e., if the 'square-free' part of n has several factors.

6.1. Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

6.2. References

Brigham E O (1974) *The Fast Fourier Transform* Prentice-Hall.

7. See Also

nag_fft_hermitian (c06ebc)
nag_conjugate_complex (c06gcc)

8. Example

This program reads in a sequence of complex data values and prints their discrete Fourier transform.

It then performs an inverse transform using nag_conjugate_complex (c06gcc) and nag_fft_complex, and prints the sequence so obtained alongside the original data values.

8.1. Program Text

```
/* nag_fft_complex(c06ecc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagc06.h>

#define NMAX 20

main()
{
    Integer j, n ;
    double x[NMAX], y[NMAX], xx[NMAX], yy[NMAX];

    Vprintf("c06ecc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    while (scanf("%ld", &n) != EOF)
        if (n > 1 && n <= NMAX)
        {
            for (j = 0; j < n; ++j)
            {
                Vscanf("%lf%lf", &x[j], &y[j]);
                xx[j] = x[j];
                yy[j] = y[j];
            }
            /* Compute transform */
            c06ecc(n, x, y, NAGERR_DEFAULT);
        }
}
```

```

Vprintf("\nComponents of discrete Fourier transform\n\n");
Vprintf("          Real          Imag\n\n");
for (j = 0; j<n; ++j)
    Vprintf("%3ld %10.5f %10.5f\n", j, x[j], y[j]);
/* Compute inverse transform */
/* Conjugate the transform */
c06gcc(n, y, NAGERR_DEFAULT);
/* Transform */
c06ecc(n, x, y, NAGERR_DEFAULT);
/* Conjugate to give inverse transform */
c06gcc(n, y, NAGERR_DEFAULT);
Vprintf("\nOriginal sequence as restored by inverse transform\n");
Vprintf("\n          Original          Restored\n");
Vprintf("          Real          Imag          Real          Imag\n\n");
for (j = 0; j<n; ++j)
    Vprintf("%3ld %10.5f %10.5f %10.5f %10.5f\n",
            j, xx[j], yy[j], x[j], y[j]);
}
else
{
    Vfprintf(stderr, "\n Invalid value of n\n");
    exit(EXIT_FAILURE);
}
exit(EXIT_SUCCESS);
}

```

8.2. Program Data

c06ecc Example Program Data

```

7
0.34907 -0.37168
0.54890 -0.35669
0.74776 -0.31175
0.94459 -0.23702
1.13850 -0.13274
1.32850  0.00074
1.51370  0.16298

```

8.3. Program Results

c06ecc Example Program Results

Components of discrete Fourier transform

	Real	Imag
0	2.48361	-0.47100
1	-0.55180	0.49684
2	-0.36711	0.09756
3	-0.28767	-0.05865
4	-0.22506	-0.17477
5	-0.14825	-0.30840
6	0.01983	-0.56496

Original sequence as restored by inverse transform

	Original		Restored	
	Real	Imag	Real	Imag
0	0.34907	-0.37168	0.34907	-0.37168
1	0.54890	-0.35669	0.54890	-0.35669
2	0.74776	-0.31175	0.74776	-0.31175
3	0.94459	-0.23702	0.94459	-0.23702
4	1.13850	-0.13274	1.13850	-0.13274
5	1.32850	0.00074	1.32850	0.00074
6	1.51370	0.16298	1.51370	0.16298